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Theory Development in Education: Implementing the ATIS Option-Set

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THEORY DEVELOPMENT IN EDUCATION

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This article is an extension of previous articles prepared by Kenneth R. Thompson, and, in particular, of "Axiomatic Theories of Intentional Systems: Methodology of Theory Construction," published in *Scientific Inquiry Journal*. These articles relate to the development of axiomatic theories of intentional systems of which education theory is one such system. The previous article on "Methodology of Theory Construction" should be read as it provides the basis for the current article. In the previous article, there is an extensive discussion of "Hypothesis-Based Research Methodologies," methodologies that pervade the social sciences. This current article presents a critique of methodologies for scientific discovery, and provides an alternative by which comprehensive, consistent, and complete theories in the social sciences, and education, in particular, can be developed. Further, it is argued that only axiomatic theories provide the means by which reliable evaluations and predictions can be obtained. A discussion of the hypothesis-driven methodologies of the social sciences is provided and why such methodologies do not result in scientific theories. Pursuant to Charles S. Peirce and subsequent confirmation by Elizabeth Steiner, theory development is the result of a reasoning process identified as *retroduction*. With this foundation, a methodology of theory construction is given that provides a means for social scientists to develop comprehensive, consistent, complete, and axiomatic theories.

A proof of the *Retroduction Theorem* is given that demonstrates that the retroductive process does result in development of new theory. Further, a proof of the *Abduction Theorem* is provided that clearly distinguishes *abduction* from *retroduction*.

Keywords: General systems theory, intentional systems, behavioral theory, education theory, SIGGS, SimEd, ATIS, A-GSBT, retroduction, abduction, theory development.

INTRODUCTION

In my, Kenneth R. Thompson's, first report, *'General System' Defined for Predictive Technologies of A-GSBT*, I briefly described the development of general systems theory and the work that led to the development of *A-GSBT*. Therein I indicated that the concept of defining *A-GSBT* as an *option set*. Such an interpretation is a distinct divergence from the prevailing interpretation of a general systems theory. This distinction will be explicated in this report.

In addition to discussing the concept of an option set that leads to an open-ended number of theories for the social sciences, I will also discuss the means by which theories are devised, which includes a discussion of retroduction and abduction. This then leads to a discussion of the process by which theories can be developed in the social sciences. This process will be distinguished from the almost universal hypothesis-driven methodology of scientific inquiry utilized in this industry. It will be shown that the use of hypotheses does not lead to theory development, but that a modification of that methodology can lead to the development of theories in the social sciences.

Examples of Hypotheses in the Social Sciences And Converting Them to Axioms

To see the distinction between hypotheses and axioms and how the latter may lead to theory, consider the following hypothesis taken from the social sciences as propounded by Tracey Clarke, Paul Ayres, and John Sweller in "The Impact of Sequencing and Prior Knowledge on Learning Mathematics through Spreadsheet Applications" (Clarke, 2005):

HYPOTHESIS: Students with a low-level knowledge of spreadsheets learn mathematics more effectively if the relevant spreadsheet skills are learned prior to attempting the mathematical tasks.

The results of testing supported this hypothesis. For our purposes the greater concern is how this hypothesis was developed and whether it may lead to theory development. The rationale for the hypothesis is stated by the researchers as follows:

> According to cognitive load theory, instruction needs to be designed in a manner that facilitates the acquisition of knowledge in long-term memory while reducing unnecessary demands on working memory. When technology is used to deliver instruction, the sequence in which students learn to use the technology and learn the relevant subject matter may have cognitive load implications, and should interact with their prior knowledge levels. An experiment, using spreadsheets to assist student learning of mathematics, indicated that for students with little knowledge of spreadsheets, sequential instruction on spreadsheets followed by mathematics instruction was superior to a concurrent presentation. These results are explained in terms of cognitive load theory. (p. 15)

The process by which this hypothesis was developed is a retroductive process; that is, cognitive load theory was used as a model to develop assertions about learning mathematics. However, there is confusion concerning this process since it is claimed: "These results are explained in terms of cognitive load theory" (see last sentence from above quotation).

If in fact the results "are explained in terms of cognitive load theory," and *cognitive load theory* (CLT) is in fact a theory, then this hypothesis may be a theorem of CLT and would be deductively obtained from that theory as a theorem to be validated; or, possibly, it is an interpretation of CLT and was derived as an abductive process—that is CLT was used as a model of mathematics learning and the content of the desired hypothesis was substituted for the content of CLT. ('Abduction' will be more fully explained later in this article, but for now it can be understood as a process similar to that in which a mathematical formula may be applied to measure or interpret an empirical event. The formula is "taken" from mathematics and "used" to help resolve a problem that concerns us. The mathematical theory from which the formula is taken does not become a part of our theory, nor does the formula become an integral part of the theory. The formula is simply a "mathematical model" that helps to answer a question.)

However, it is not claimed that either approach was used, so the question moves to whether or not CLT is actually a theory, or is it a hypothesis that has been validated through various tests? If it is a theory, then we may be able to determine in what sense it is claimed that CLT "explains" the hypothesis.

Before proceeding, we need to clarify various types of 'theory'. In particular, we are concerned with the distinctions between explanatory and descriptive theories.

Explanatory Theory. An explanatory theory is an explanation of an observed phenomenon that sets forth the causal relations that result in what is observed.

Descriptive Theory. A descriptive theory defines an observed phenomenon in terms of its component parts as descriptive of its properties, and defines the interactions of those component parts that result in the observed outcome.

First, the use of the term *theory* in the context of CLT indicates that it is not either a formal theory or an axiomatic theory. If it is a theory, then it appears to be a descriptive theory. But, even as a descriptive theory, it appears to be very limited in scope and functions more like a hypothesis since deductive derivations are difficult to obtain. But, let us look at this more carefully.

Cognitive Load Theory, developed by J. Sweller, is founded on the following four principles:

- Working memory, or short-term memory, has a maximum capacity identified as *maximum cognitive load*
- Information that exceeds *maximum cognitive load* is lost
- Learning requires that *cognitive load* remain below some value that is less than *maximum cognitive load*
- Long-term memory is consciously processed through working memory

NOTE: The "theory" and "axioms" presented below are for the sole purpose of demonstrating a possible construction of an axiomatic theory and in no way is to be construed as a replacement for the *Cognitive Load Theory* developed by J. Sweller. In fact, it is only as a result of the careful development of CLT that it is possible to derive an axiomatic theory therefrom. In general, it is very difficult to ascertain axioms from a descriptive theory, since they frequently are so vaguely developed that explicit statements of their assumptions are difficult or impossible to determine. Fortunately, CLT is not such a theory.

Provided below is a preliminary development for a *Theory of Memory and Learning* that is retroductively-derived from CLT. Briefly presented are the primitive terms, initial axioms, definitions and a few theorems of the theory.

Theory of Memory and Learning

PRIMITIVE TERMS: *Cognition*, *memory*, *working memory*, *consciously*, *cognitive load, mental structures*, *patterns*, and *languages*

AXIOM 1: Working memory is that memory which is used to consciously process information.

AXIOM 2: Working memory has a maximum capacity identified as *maximum cognitive load*.

AXIOM 3: Working memory that is maintained below *maximum cognitive load* results in short-term memory acquisition.

AXIOM 4: Long-term memory is consciously processed through working memory.

AXIOM 5: Cognition is determined by a sequence of recognizable patterns or languages.

AXIOM 6: Long-term memory is short-term memory that is processed and related to an existing or newly developed cognitive schema, or structure.

DEFINITION 1: 'Cognitive schemas' are memory constructs that map short-term memory cognition onto devised mental structures that interpret immediate cognition.

DEFINITION 2: 'Learning' is defined as that processed cognitive load that results in the acquisition of short-term memory.

As a result of these axioms and definitions, we obtain the following theorems:

THEOREM 1: Information that exceeds *maximum cognitive load* is not cognizable.

PROOF OF THEOREM 1:

- Working memory has a maximum capacity identified as *maximum cognitive load*. (Axiom 2.)
- That which exceeds maximum capacity is not cognizable. (Definition of 'maximum'.)

Another way of stating Theorem 1 is: Information that exceeds *maximum cognitive load* is lost—that is, the second statement of the four principles cited above for CLT.

THEOREM 2: For learning to occur, *cognitive load* must remain below *maximum cognitive load*.

PROOF OF THEOREM 2:

- Cognitive load that exceeds maximum cognitive load is not cognizable and, therefore, not processed. (Theorem 1.)
- Working memory that is maintained below *maximum cognitive load* results in short-term memory acquisition. (Axiom 2.)
- Short-term memory acquisition results in learning. (Definition of 'learning'.)

Now the problem is to determine if the hypothesis relating to learning mathematics considered previously can be derived from this theory. Stating the hypothesis again:

HYPOTHESIS / THEOREM 3: Students with a low-level knowledge of spreadsheets learn mathematics more effectively if the relevant spreadsheet skills are learned prior to attempting the mathematical tasks.

PROOF OF THEOREM 3:

- Students do not have cognitive schemas relating to spreadsheets. (Assumption of Theorem 3.)
- Students do not have cognitive schemas relating to mathematics. (Assumption of Theorem 3.)
- Lack of cognitive schemas precludes long-term memory. (Axiom 6.)
- Spreadsheet cognition precedes mathematics cognition. (By Axiom 5 and assumption of Theorem 3, the spreadsheet structure provides the basic "language" by which mathematics is learned.)
- A spreadsheet cognitive schema must be developed for long-term memory to take place. (Axiom 6.)
- Therefore, relevant spreadsheet skills must be learned prior to the learning of mathematical tasks. (Conclusion of Theorem 3.)

The importance of this axiomatic development is that now a much stronger claim can be made concerning Theorem 3. Whereas the initial researchers could only claim: "These results are explained in terms of cognitive load theory," it can now be claimed more strongly: "These results are deductively obtained from the *Theory of Memory and Learning* and are validated by empirical testing."

But what is the far-reaching effect of this second approach? By validating Theorem 3 the researchers have not only validated their "hypothesis," but have now provided support for the theory. This validation has now initiated a process that, hopefully, will eventually provide a "preponderance of evidence" that the theory consistently provides valid outcomes. By framing CLT as an axiomatic theory, every validation of a theorem (or hypothesis if you want) validates not only the theorem but the theory. Eventually, we will be able to obtain theorems deductively from the theory and proceed with confidence that the outcome is accurate, with or without further validation. This is very important, since otherwise every hypothesis must be continually validated in every new setting, in every new school, in every new learning environment.

Whereas the hypothesis has been validated for this one group of students learning mathematics from a spreadsheet, what can we say if instead of a spreadsheet, new computer software is utilized? Will they have to learn the software before learning the mathematics? At first glance, the answer should be "obvious" even without any testing. But, for the sake of making a point, the point is also "obvious"—we have already provided the proof that they would have to learn the software and we do not have to, once again, conduct testing to validate the theorem.

But now, what about results that are not so obvious?

Theory of Memory and Learning—Applications

Theorem 3 provided content that is not stated in the theory axioms. Applications of this theory are the result of the logical process of abduction; that is, the theory content is determined independent of the theory and substituted for the theoretical constructs. For example, "working memory" of the theory is replaced by "spreadsheet" and "mathematics" by the specific application. Additional theory applications can be obtained by interpreting various cognitive schemas.

For a non-obvious theory outcome, consider the following schemas that have been established as part of long-term memory: (1) Learned behavior described as the schema "assertive"; and (2) Learned behavior described as the schema "attention-to-detail."

When students learn to keyboard it is frequently asserted that in order to improve speed and accuracy they must practice keyboarding. However, if that were accurate, then anyone who has been keyboarding for many years should be doing so at approximately 60 words-per-minute with great accuracy—whereas this is not the case. There must be more to developing speed and accuracy than practicing keyboarding. From the *Theory of Memory and Learning* it is determined that these students have developed certain cognitive schemas defined as *assertive* and *attention-to-detail* that are independent of content.

As a result of these cognitive schemas and the process of abduction by which theory content is determined, the following theorem is obtained:

THEOREM 4: Keyboarding speed can be improved by any off-task activity that increases one's *assertiveness*; and keyboarding accuracy can be improved by any off-task activity that increases one's *attention-to-detail*.

Theorem 4 is a direct result of Axiom 6 and Axiom 5. Theorem 4 is a non-obvious result of the *Theory of Memory and Learning* that was derived from *Cognitive Load Theory*. While there is anecdotal evidence that Theorem 4 is valid, actual validation or refutation of Theorem 4 is left to those who are more skilled at constructing appropriate tests. Whether Theorem 4 is found to be valid or not, the efficacy of an axiomatic theory has been demonstrated as being one that results in non-obvious conclusions. And, as seen here, an axiomatic theory does not have to be formal, although the formalization of this theory may result in conclusions that are even more unexpected.

The next step would be to make the theory more comprehensive. Several theories may be related to CLT such as the *Information Processing Theory* by G. Miller (Miller, 1956), *Human Memory Theory* by A.D. Baddeley, *Cognitive Principles of Multimedia Learning* by R. Moreno and R.E. Mayer (Moreno, 1999), *Parallel Instruction Theory* by R. Min (Min, 1992), *Anchored Instruction* by J.D. Bransford (Bransford, 1990), and *Social Development Theory* by J.V. Wertsch (Wertsch, 1985), among others. All of the relevant theories could be brought under one theory that provides the first principles from which all others are derived. Then, all validations further not only the specific research but provide greater confidence in the theory that is founded on the first principles. Such unification would provide a basis by which the unexpected may actually be determined rather than describing that which is already recognized.

THE RETRODUCTION AND ABDUCTION THEOREMS

As previously stated, theory development, or emendation, is accomplished by a process of retroduction. Theory extension is accomplished by deduction or abduction. In mathematical logic, the deduction theorem is of significant importance. The deduction theorem states that if we have a conclusionary statement that is derived from other statements that are axioms, assumptions, or derived from axioms by Modus Ponens, then we can assert that such statements imply the conclusionary statement. Formally, the deduction theorem is stated as follows:

DEDUCTION THEOREM: If P $\vdash$ Q, then $\vdash$ P $\supset$ Q. That is, if P yields Q, then yields P implies Q.

Neither the retroduction theorem nor the abduction theorem is required for mathematical logic. However, for theory development in the social sciences, both are required and they must be carefully distinguished. As stated initially, it should be clear that 'retroduction' and 'abduction' are not the same, and that they have been equated only because of "corrupted text."

So, what is the formal distinction between 'retroduction' and 'abduction'? Consider the following example.

A *direct affect relation* in a behavioral topological space is defined in terms of a *mathematical vector*. That is, it is recognized that the concept of *vector* is applicable to this behavioral theory. This transition was recognized as a result of affect relations being interpreted as "force fields." That is, gravitational and electromagnetic force fields are vector fields; fluid velocity vectors, whether in the ocean or the atmosphere, are vector fields; weather pressure gradients are vector fields; and affect relations of an intentional system are vector fields—they are dynamic. They exhibit both direction and magnitude. They exhibit the change and flow of any other empirical vector field.

This process of applying an interpretation to the mathematical construct *vector* is a logical process of *abduction*. This is not a process of "moving backward," but a process of "taking from." The mathematical measure is simply being applied to the content of a behavioral theory. There is no theory development; there is simply an explication of the theory by mathematical means. The important concept here, as defined by Steiner, is that there is no "theory development," but "theory explication," the logical process normally the result of deduction. The mathematical concept of a vector field is utilized as a measure to further explicate the theory. The concept *affect relation* was already in the theory, so it is clear that no theory development was accomplished. Was there a retroduction of the "form" as a single predicate from mathematics? No. What is being utilized here is simply the definition of a vector field. The definition of *vector field* is being "taken from" mathematics in order to deductively explicate the theory of affect relations.

This research defines 'retroduction' and 'abduction' as distinct logical processes, and such that they complement the logical process of deduction and induction as follows:

- ***Retroduction*** is the logical process by which a point of view is utilized to devise a conjecture or theory.
- ***Deduction*** is the logical process by which a conclusion is obtained as the implication of assumptions.
- ***Abduction*** is the logical process by which a theoretical construct of one theory is utilized to analyze or interpret the parameters of another theory.
- ***Induction*** is the logical process by which theory is evaluated.

While the *Deduction Theorem* is a standard part of mathematical logic, this research extends this analysis to include the *Retroduction Theorem* and *Abduction Theorem*.

Steiner defines retroduction schematically as follows:

> Given theories $\mathcal{A}$ and $\mathcal{B}$, theory $\mathcal{A}$ is a devising model for theory $\mathcal{B}$ if there is a subset, $\mathcal{C}$, of $\mathcal{A}$ such that the predicates of $\mathcal{B}$ are a representation in substance or form of the predicates of $\mathcal{C}$; whatever is true of $\mathcal{C}$ is true of $\mathcal{B}$; and whatever is true of $\mathcal{B}$ is not true of $\mathcal{A}$.

Initially it would appear that the following implication holds: $\mathcal{A} \supset \mathcal{B}$. However, as Steiner points out, "The theory or conjecture that emerges (conclusion) contains more than the theory or point of view from which it emerges (premises). The implication, then, can only hold from the conclusion to the premise"; that is, $\mathcal{B} \supset \mathcal{A}$. It could be argued that the sentential and predicate logic do not hold in this instance. But, if not, we are left with a state of confusion when we are attempting to develop a scientific theory that relies on just such logics. Therefore, it must be assumed that the logic holds and we need to take a closer look at just what is required.

Taking retroduction, as it is conceptually defined; we have that Theory $\mathcal{A}$ is a devising model for Theory $\mathcal{B}$. By this is meant that the predicates for Theory $\mathcal{B}$ are derived as representations from a subset, $\mathcal{C}$, of the predicates of Theory $\mathcal{A}$; that is:

$$\mathcal{P}(\hat{h}) \in \mathcal{B} \supset \mathcal{P}(h) \in \mathcal{C} \subset \mathcal{A}.$$

But also we have that Theory $\mathcal{B}$ results in more than what was in Theory $\mathcal{A}$; since, otherwise, it would not be an emendation of Theory $\mathcal{A}$, but simply a replication. This emendation of Theory $\mathcal{A}$ that results in more than what is in $\mathcal{A}$ is formally defined as:

$$\exists \mathcal{P}(\hat{h}) \in \mathcal{B}[\forall \mathcal{P}(h) \in \mathcal{A}[\sim(\mathcal{P}(\hat{h}) \in \mathcal{B} \stackrel{?}{=} \mathcal{P}(h) \in \mathcal{A}];$$

where '∃' is read "there exists", '~' is read "not" or "it is not the case that", '∀' is read "for all", and '$\stackrel{?}{=}$' is read "is isostruct to"; and isostructism is a mapping of one entity to another to which it is isomorphic or isosubstantive. (That is, "There exists a Predicate $\hat{h}$, an element of $\mathcal{B}$, such that, it is not the case that the Predicate $\hat{h}$ is an element of $\mathcal{B}$ is isostruct to Predicate h an element of $\mathcal{A}$.) This meets the final requirement by Steiner.

. The formal representation of 'isostruct' is:

$$\mathcal{P}(\hat{h}) \in \mathcal{B} \stackrel{?}{=} \mathcal{P}(h) \in \mathcal{C} =_{df} \mathcal{P}(\hat{h}) \in \mathcal{B} \cong \mathcal{P}(h) \in \mathcal{C} :\vee: \mathcal{B}(\hat{h}) \in \mathcal{B} \approxeq \mathcal{B}(h) \in \mathcal{C};$$

where, '≅' is read "is isomorphic to" and '≊', "is isosubstantive to".

Essentially, a theory is retroductively emended from an existing theory by devising predicates of the existing theory to represent concepts desired in the new theory; and as a result, the new theory will contain additional deductively-derived predicates that are not part of the existing theory—the new theory contains more than what was derived from the exiting theory.

An isostructism is a mapping of one entity to another to which it is isomorphic or isosubstantive. An isomorphism is a mapping of one entity into another having the same elemental structure, whereby the behaviors of the two entities are identically describable by their affect relations. An isosubstantism is a mapping of one entity into another having similar predicate descriptors.

For example, in the devised *Theory of Memory and Learning* (TML): (1) The CLT component-relations of short-term and long-term memory are isomorphic to the component-relations in TML; (2) TML *cognitive schemas* are represented as morphisms that are isomorphic to the CLT construct *schema*; and (3) Axioms 1 to 6 are isosubstantive to the four principles of CLT or parts thereof, such that the affect relations of both describe similar behaviors. Further, Theorem 4 is deductively derived from Axioms 5 and 6 and results in a predicate of TML that is not contained in CLT.

The *Retroduction Theorem* is formalized as follows:

RETRODUCTION THEOREM:

If $\{\mathcal{P}(\hat{h}) \mid \mathcal{P}(\hat{h})\in\mathcal{B}\}$ is a set of predicates that comprise Theory $\mathcal{B}$, then:

$$\exists\mathcal{P}(\hat{h})\in\mathcal{B}[\forall\mathcal{P}(h)\in\mathcal{A}[\sim(\mathcal{P}(\hat{h})\in\mathcal{B} \doteq \mathcal{P}(h)\in\mathcal{A}],$$

$$\mathcal{P}(h)\in\mathcal{C}\subset\mathcal{A} \stackrel{?}{\doteq} \mathcal{P}(\hat{h})\in\mathcal{B},$$

$$\exists\mathcal{P}(\hat{h})\in\mathcal{B}[\sim(\mathcal{P}(\hat{h})\in\mathcal{B} \stackrel{?}{\doteq} \mathcal{P}(h)\in\mathcal{A})] \vdash$$

$$\mathcal{P}(\hat{h})\in\mathcal{B} \supset \mathcal{P}(h)\in\mathcal{C}\subset\mathcal{A}.$$

PROOF OF RETRODUCTION THEOREM:

For the purposes of this proof, since the conclusion is simply the result of the assumptions by definition, all that needs to be argued is that the *Predicate Calculus* applies to Theory $\mathcal{B}$. To apply, Theories $\mathcal{A}$ and $\mathcal{B}$ must be isostruct with respect to $\mathcal{C}$. By assumption, they are. All we have to show is that $\mathcal{P}(\hat{h})\in\mathcal{B}$ represents a consistent set of predicates that have been derived from Theory $\mathcal{A}$ and that they make $\mathcal{B}$ a theory.

$\mathcal{P}(h)\in\mathcal{A}$	Assumption
$\mathcal{P}(\hat{h})\in\mathcal{B} \supset \mathcal{P}(h)\in\mathcal{C}\subset\mathcal{A}$	Assumption
$\exists\mathcal{P}(\hat{h})\in\mathcal{B}\ \forall\mathcal{P}(h)\in A[\sim(\mathcal{P}(\hat{h}) \supset \mathcal{P}(h))]$	Assumption
$\mathcal{P}(\hat{h})\in\mathcal{B}$ are derived from $\mathcal{P}(h)\in\mathcal{C}\subset\mathcal{A}$	Assumption

All we now need to demonstrate is that $\mathcal{P}(\hat{h})\in\mathcal{B}$ is a consistent theory.

If $\{(\hat{w},\hat{y})\in\mathcal{B}\times\mathcal{B} \mid \mathcal{P}(\hat{w},\hat{y})\} \cong \{(w,y)\in\mathcal{A}\times\mathcal{A} \mid \mathcal{P}(w,y)\}$, then all of the consistent logical conclusions relating to $\mathcal{P}(w,y)$ also apply to $\mathcal{P}(\hat{w},\hat{y})$, by substitution.

If $\mathcal{P}(\hat{h}) \cong \mathcal{P}(h)$, then any component of $\mathcal{A}$ that satisfies $\mathcal{P}(h)$ has a corresponding component in $\mathcal{B}$ that satisfies $\mathcal{P}(\hat{h})$.

Therefore, the components, relations, and predicates which are valid for Theory $\mathcal{A}$ have corresponding components, relations, and predicates in $\mathcal{B}$, resulting in the consistency of $\mathcal{B}$. By definition, the predicates of $\mathcal{B}$ comprise a theory.

Essentially, since the theories are isostruct, any proof in $\mathcal{C}$ is applicable to a corresponding proof in $\mathcal{B}$, since they will have corresponding axioms and assumptions. Further, any predicate in $\mathcal{B}$ not in $\mathcal{A}$ can be taken as an assumption or axiom from which resulting theorems can be derived by the *Predicate Calculus* or the predicate is deductively derived from the existing predicates of $\mathcal{B}$.

The difficulty in applying the retroduction theorem is in establishing that the two predicate sets are in fact isostruct and represent a viable theory. The "viable theory," of course, is established by its empirical validations.

The *Abduction Theorem* will now be proved.

ABDUCTION THEOREM:

Given Theories $\mathcal{A}$ and $\mathcal{B}$, a subset, $\mathcal{C}$, of Theory $\mathcal{A}$ is a formal model-of a subset, $\mathcal{G}$, of Theory $\mathcal{B}$ if the predicates of $\mathcal{G}$ are an equivalent representation in form of the predicates of $\mathcal{C}$; whatever is true of $\mathcal{C}$ is true of $\mathcal{G}$; and whatever is true of $\mathcal{G}$ is true of $\mathcal{C}$. The formal statement of the Abduction Theorem is:

$$\mathcal{P}(h) \cong \mathcal{P}(\hat{h}) \vdash \mathcal{P}(h)\in\mathcal{C}\subset\mathcal{A} :\equiv: \mathcal{P}(\hat{h})\in\mathcal{G}\subset\mathcal{B}$$

:

$$h \equiv \hat{h},\ \mathcal{P}(h) \cong \mathcal{P}(\hat{h}),\ \mathcal{P}(h) \cong \mathcal{P}(\hat{h}) \vdash \mathcal{P}(h) \in \mathcal{C} \subset \mathcal{A} :\equiv: \mathcal{P}(\hat{h}) \in \mathcal{G} \subset \mathcal{B}$$

PROOF OF ABDUCTION THEOREM:

(1)	$h \equiv \hat{h}$	Assumption
(2)	$\mathcal{P}(h) \cong \mathcal{P}(\hat{h})$	Assumption
(3)	$\mathcal{P}(h_1), \mathcal{P}(h_2), \ldots, \mathcal{P}(h_n) \in \mathcal{C} \subset \mathcal{A}$	Assumption
(4)	$\mathcal{P}(\hat{h}_1), \mathcal{P}(\hat{h}_2), \ldots, \mathcal{P}(\hat{h}_n) \in \mathcal{G} \subset \mathcal{B}$	Substitution, 1 in 3
(5)	$\therefore \mathcal{P}(h_1), \mathcal{P}(h_2), \ldots, \mathcal{P}(h_n) \in \mathcal{C} \vdash \mathcal{P}(\hat{h}_1), \mathcal{P}(\hat{h}_2), \ldots, \mathcal{P}(\hat{h}_n) \in \mathcal{G}$	From 3 and 4
(6)	$\vdash \mathcal{P}(h_1), \mathcal{P}(h_2), \ldots, \mathcal{P}(h_n) \in \mathcal{C} \supset \mathcal{P}(\hat{h}_1), \mathcal{P}(\hat{h}_2), \ldots, \mathcal{P}(\hat{h}_n) \in \mathcal{G}$	Deduction Theorem on 5
(7)	$\therefore \mathcal{P}(\hat{h}_1), \mathcal{P}(\hat{h}_2), \ldots, \mathcal{P}(\hat{h}_n) \in \mathcal{B} \vdash \mathcal{P}(h_1), \mathcal{P}(h_2), \ldots, \mathcal{P}(h_n) \in \mathcal{C} \subset \mathcal{A}$	From 4 and 3
(8)	$\vdash \mathcal{P}(\hat{h}_1), \mathcal{P}(\hat{h}_2), \ldots, \mathcal{P}(\hat{h}_n) \in \mathcal{B} \supset \mathcal{P}(h_1), \mathcal{P}(h_2), \ldots, \mathcal{P}(h_n) \in \mathcal{C} \subset \mathcal{A}$	Deduction Theorem on 7
(9)	$\mathcal{P}(h) \cong \mathcal{P}(\hat{h}) \vdash \mathcal{P}(h_1), \ldots, \mathcal{P}(h_n) \in \mathcal{C} \subset \mathcal{A} :\equiv: \mathcal{P}(\hat{h}_1), \ldots, \mathcal{P}(\hat{h}_n) \in \mathcal{B}$	Definition, 4 & 7 / Q.E.D.

The significance of this theorem is that formal predicates of a given theory that are isosubstantive to formal predicates of another theory, define the properties of the second theory. For example, as noted earlier, mathematical vectors can be used to analyze gravitational and electromagnetic force fields; fluid velocity, whether in the ocean or the atmosphere; weather pressure gradients; and affect relations of an intentional system. The mathematical formulas that are used for these analyses are not changed when they are utilized to determine values within any of these applications—they are simply used to analyze various systems and obtain desired outcomes.

IMPLEMENTING THE *ATIS OPTION SET*

In the preceding sections the necessity of an axiomatic theory has been argued. In addition, it has been shown how a hypothesis-driven methodology can be modified to develop an axiomatic theory. And the place of abduction as a logical process of theory extension has been demonstrated along with providing proofs of the retroduction and abduction theorems.

In this section, a methodology of theory construction from the *ATIS Option Set* will be discussed.

Although only *general system* as defined for *ATIS* has been presented, it will be instructive to see how a theory is developed for an empirical system once a comprehensive *ATIS* theory is obtained. This will assist in understanding the stages of development of *ATIS* as they are presented in future articles in this journal.

Essentially, there is a five-step process for implementing the *ATIS Option Set*:

1. Identify the problem-statement that defines the components of the empirical system
2. Identify the affect relations of the target system
3. Analyze the affect relations to determine relevant properties
4. The relevant properties identify the related axioms
5. From the related axioms, derive the theory-predicted outcomes

While this process may appear to be simple, it is anything but. The first, and possibly the most difficult part of the process is the first step. The following example will help to explain the process.

EXAMPLE OF *ATIS*-APPLICATION STEPS:

Step 1: Identify the problem-statement that defines the empirical system

For our example we will use a research project from education that was designed to determine the extent to which student independence in a classroom improves student academic performance. (While

ATIS has not yet been presented, the following discussion indicates various properties and qualifiers that are a part of *ATIS*. As this is but a demonstration of the procedure for developing an axiomatic theory, such terms are to be taken at face value, as the actual presentation of the theory will be part of a future article.) The initial statement of the problem was as follows:

PROBLEM STATEMENT: How can teachers foster autonomy and intrinsic motivation for learning in a classroom system?

When working with a well-defined theory model like ATIS, we must always be careful to re-read the definitions of our terms. 'Autonomy' is defined as a "system that is component closed." I do not believe that we want this, since no learning could then take place. Frequently, some of our terms will have different meanings in the common language from that of our formal language, as is the case here. Frequently, 'autonomy' and 'independence' are used synonymously in the common language, whereas they have very distinctly different meanings in the formal language. What we want is 'independence'. And, we want the students to be characterized by independence. So, let us restate our problem statement:

PROBLEM STATEMENT—REVISION 1: How can teachers foster student independence and intrinsic motivation for learning in a classroom system?

Now let us consider the use of the term 'intrinsic'. 'Intrinsic' means an "essential or inherent" part of something. If "motivation" is an "essential or inherent" part of a student's learning, then there is nothing to develop, since they will already have it. Possibly what is meant is that the students are to develop dispositional behaviors that characterize self-motivational learning. Further, 'intrinsic motivation' is not part of the formal language of *ATIS*, so we would either have to define it or choose something different. For now, I believe that what is meant is the development of the students' dispositional behavior; *dispositional-behavior* is a part of the *ATIS* formal language.

Further, one of the *Behavioral Affect Relation Qualifiers* for *ATIS* is "Inquiry." So, we might want to make the problem statement as follows:

PROBLEM STATEMENT—REVISION 2: How can teachers foster student independence and develop student inquiry dispositional behavior for learning in a classroom system?

Now let's consider what teachers will actually do. Will they actually "foster" student behavior, or "facilitate" student behavior? For now, I will opt for "facilitating" since we already have a *Facilitating Behavioral Affect Relation Qualifier* in *ATIS*. If the term 'foster' is desired, then it will have to be defined within the formal theory. So, our problem statement is as follows:

PROBLEM STATEMENT—REVISION 3: How can teachers facilitate student independence and develop student inquiry dispositional behavior for learning in a classroom system?

However, we now need to change this from a question to a statement; since we must have a problem "statement." A theory describes a system; it does not ask questions about a system. The initial assertion was that student independence would result in greater learning, and that seems to be the intent of the current proposed question. So, combining the two, and after some further iteration, the problem statement is as follows:

PROBLEM STATEMENT—REVISION 4: The system is a Family of Student-Independent Systems such that each Student-Independent System is characterized by student-inquiry dispositional behavior affect relations; the negasystem is composed of, in part, a Family of Instructor-Facilitating Systems; and there exists an Instructional System Morphism between the two families of systems.

With this problem statement, we must define *Instructional System Morphism*, as follows:

Instructional System Morphism $=_{df}$ a relation between two systems that are related by *Instructional Affect Relations*, where one system is identified as a *Family of Instructor Components*, and the other system is identified as a *Family of Student Components*.

Now we have defined a system that can be analyzed—whether or not such system will actually result in greater student learning remains to be seen. But, we can now apply various properties and their related axioms to determine what outcomes would be expected as a result of this system structure. This analysis will also answer how teachers can structure their classrooms so that the outcomes can be obtained. That is, the system structure will be presented, and it is up to the teacher to assure that structure in the classroom. If teachers have difficulty "letting go" of their traditional role as “controller and director of classroom activities,” such reluctance is irrelevant to the analysis of the system. If they do not "let go," then they will not realize the benefits of the system—if such design does in fact result in greater student learning.

Implementing the system design is the same problem that engineers have when they implement an architectural design. If they refuse to work with marble or cannot get the marble that is called for in the architectural design, then the outcome will not reflect the initial design. The same is true for the teacher and the classroom. But, such lack of implementation has nothing to do with the analysis of the system.

This is exactly why one must always define the system without preconceptions of what is possible, or in any way prejudice the outcomes by defining the outcomes in the initial problem statement. Just define your system with a properly-worded problem statement and see what happens.

Step 2: Identify the affect relations of the target system

In order to determine the properties of a system and then the applicable axioms and theorems, we will need to know very specifically what the components are and their relatedness. These are derived from the initial problem statement.

To facilitate the identification of the components and their relatedness, it is sometimes beneficial to construct a diagram that identifies all of the various system partitions, components and relatedness. To start, we will design a system that has only one instructor-system and 3 student-systems. Also, we must be sure to clearly define the "universe." From the perspective of the family of students, the instructor-system is in the negasystem. The negasystem will also contain a variety of community-based organizations, the students' families, etc. As a comparison, it will be instructive to diagram first the traditional classroom teaching structure as shown in Diagram 1. The dotted-line encompasses the traditional classroom consisting of the teacher, students and facilities for learning.

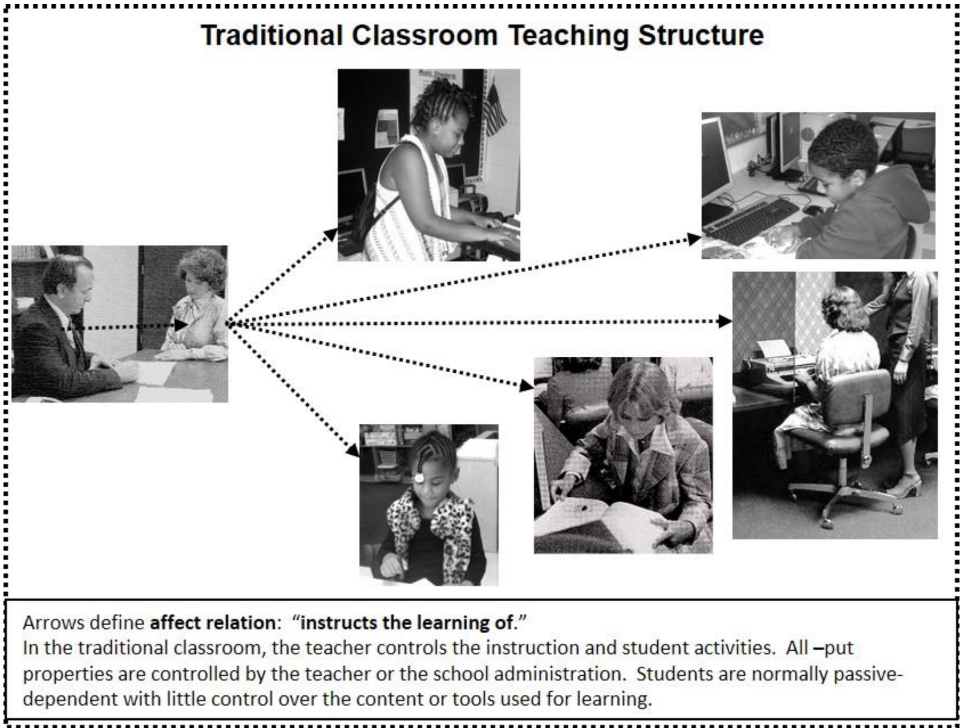


Diagram 1

Now, the system with which we are currently concerned might be diagramed as shown in Diagram 2. But, whereas Diagram 2 indicates only the main connectedness desired, Diagram 3 indicates the complexity of the target system that we actually have.

The target system is the family of student-components as indicated by the rectangle surrounding the three students.

Everything within the diagram outside the family of student-components is in the negasystem.

The complexity of the analysis is increased by the fact that both the student-component system and the teacher-component system have all of the –put properties that will be of concern for this analysis. Further, each student-component has all of the –put properties due to their own individual relations to the teacher-component and community.

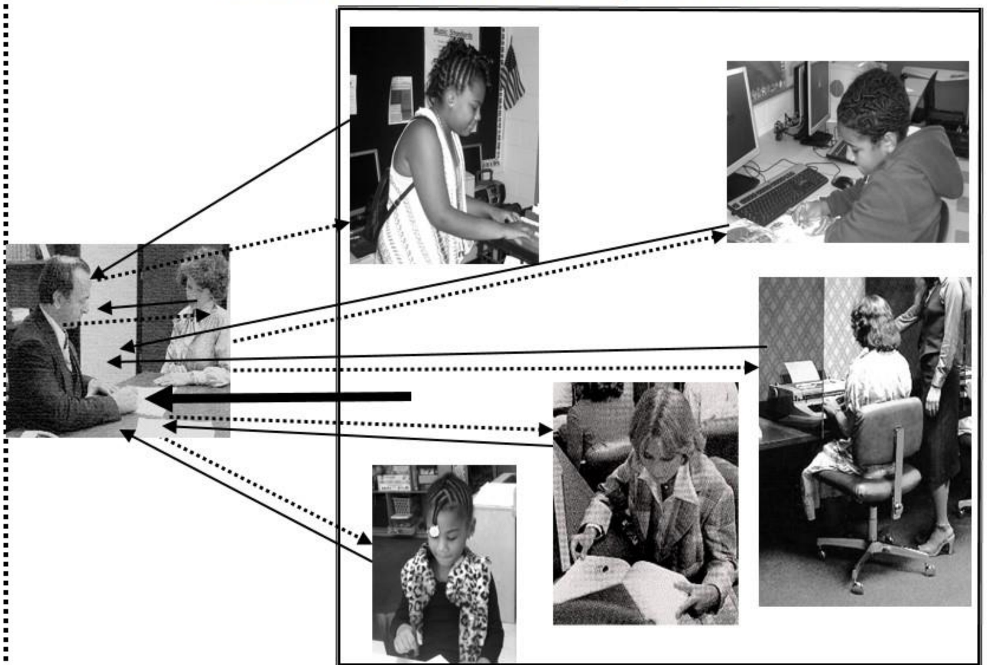
Having obtained an initial representation of the complexity of the system, we can now specify the various "relations" that occur within the system.

As described in the caption to the diagram, the three main affect relations are:

- ♦ "**Requests course information from**,"
- ♦ "**Requests individual instructional guidance from,**" and
- ♦ "**Provides requested instructional guidance to**."

In addition, there are numerous affect relations from the Family of Student-Independent Systems to the social community and from each student individually to this social community. Further, the first two main affect relations cited here will result in numerous affect relations that further the leaning of each student as they take initiative to achieve the learning that they have determined for themselves.

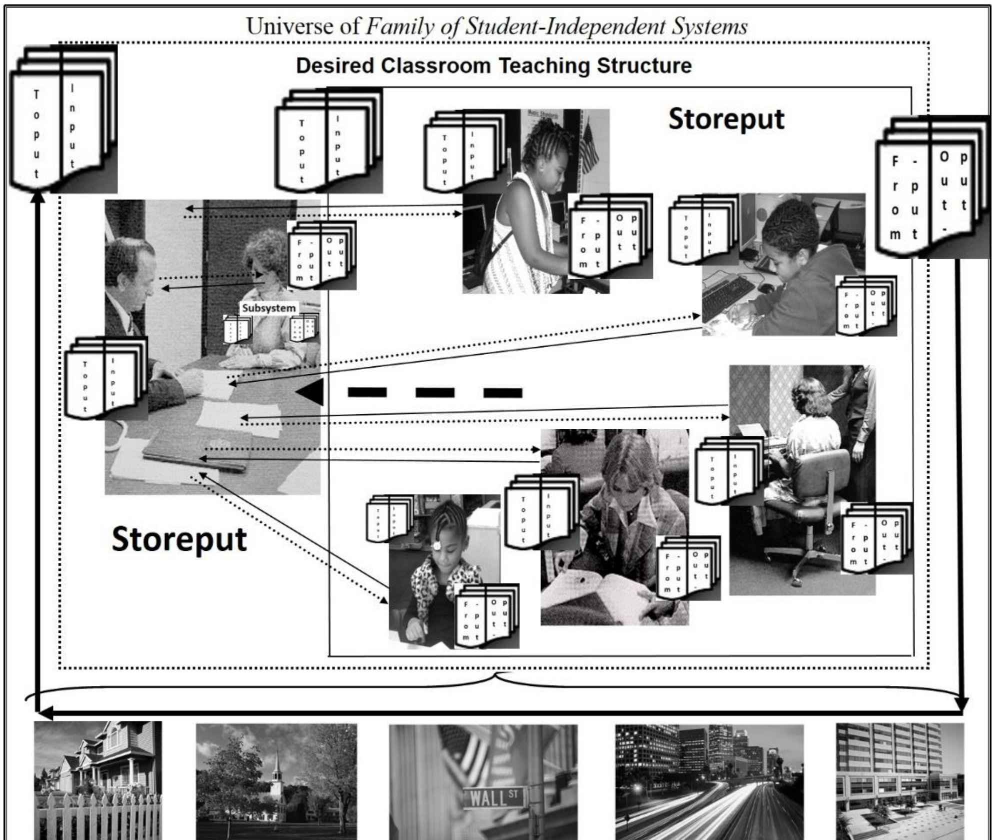
Desired Classroom Teaching Structure



The heavy arrow defines *Family of Student-Component Systems* **affect relation**: “**seeks course and content structure from**.”
The solid arrows define *Student-Component System* **affect relation**: “**seeks individual instructional guidance from**.”
The dashed arrows define *Teacher-Component System* **affect relation**: “**provides requested instructional guidance to**.”
In this classroom, the students control their own learning and activities. All –put properties are controlled by the students both as a family and individually. Students are independent and function as active-dependent individuals with significant control over the content and tools used for learning. While the dotted-line surrounding this diagram indicates that the teacher and students are in the same physical classroom, the outline around the students indicate that they will be viewed as part of the target system, the *Family of Student-Component Systems*.

Diagram 2. The teacher is in the negasystem, and Diagram 3 will more clearly define this relation as well as indicate additional components of the negasystem.

Family of Student-Independent Systems[1]



Community-Based Support Organizations for *Student-Component Systems*. For example:
Students' family and homes, religious institutions, financial institutions, transportation systems and automotive dealers, and all other organizations and businesses which a student may require in the pursuit of education outcomes.

[1]**Legend:** The symbol, ❒, indicates the "toput-input" and "fromput-output" partitions for each system.

Note that the student with the teacher is considered a subsystem of the teacher's system.
The dotted line surrounding the *Family of Student-Component Systems* and the *Family of Teacher-Component Systems* indicates the structure of a traditional classroom.

Storeput, as indicated for the *Student-Component Systems* is that portion of the system contained between input and fromput of each system.

The **heavy dashed arrow** from the *Family of Student-Component Systems* is **affect relation**: "***Seeks course and content structure from***" both from the *Family* and each individual *Student-Component System*.
The **light solid arrows** are **affect relations**: "**seeks individual instructional guidance from**."
The **light dashed arrows** are **affect relations**: "**provides requested instructional guidance to**."
The **heavy solid arrows** flowing from the *Classroom Structure Output* through the *Community-Based Support Organizations* and back to the *Classroom Structure Toput* indicate the **affect relations** that are modified as a result of this **Feedback**.

Diagram 3

With respect to the affect relations, we will start by considering the following:

- Student-Family requests instructional assistance from
  - ♦ Student-Family requests course information from
  - ♦ Student-Family requests learning tools from
- Student requests instructional assistance from
  - ♦ Student requests critique of academic work from
  - ♦ Student requests research assistance from
- Instructor provides requested assistance to
- Student utilizes learning tools
  - ♦ utilizes computer systems
  - ♦ utilizes text and resource books
  - ♦ utilizes library facilities
- Student relies on community services
  - ♦ relates to home and family
  - ♦ relates to religious institutions
  - ♦ relates to financial institutions
  - ♦ relates to transportation systems
  - ♦ relates to business establishments

While we have defined various affect relations relating to the learning of the students, these affect relations do not yet tell us anything about the effect of "control" on the learning outcomes of the students; that is, whether or not *student-independentness* results in effective learning. To determine outcomes relating to student-independentness, we determine various *control connectedness properties* and analyze the results with respect to this system. It is assumed that the following properties, each of which will be determined or measured for its degree of applicability to the system, influence *control* and *student-independentness*:

Active Dependentness,
Passive Dependentness,
Centralizationness,
Compactness,
Derived production outputness (which is *learning*),
Dispositional behaviorness,
Efficientness,
[All of the feedness- functions],
Filtrationness,
[All of the -putness properties],
Heterarchicalness,
Hierarchicalness,
Independentness,
Interdependentness,
Regulationness,
Spillageness,
Stableness,
Steadiness,
Strainness,
Stressness,
Strongness,
Unilateralness,
Vulnerableness,
Weakness, and
Wholeness.

Since an affect relation is a set of connected components, it can be analyzed with respect to each of the above listed properties since component-connectedness defines each property. Also, since each of the affect relations identified above can be analyzed with respect to all of these properties, the magnitude of the task becomes apparent.

However, regardless of the complexity, the main advantage of this methodology is that it can actually be done, since it has been reduced to a logic evaluation. A conventional mathematical model would be difficult if not impossible to use, since the equations for such relatedness are not known. (For example, what is the mathematical model for "Student-A requests instructional assistance from Teacher-M"?) It is suggested, however, that a topological vector analysis may be applicable and will be required to obtain real-time outcome predictions. The applicability of topological analyses will be considered in a future article.

The next step for now is to determine how to unify all of the outcomes relating to each affect relation.

First, to minimize the task, the affect relations listed above can be reduced to their main categories, and then we will only be concerned with the first three of those categories:

- Student-Family requests instructional assistance from
- Student requests instructional assistance from
- Instructor provides requested assistance to

This analysis will be considered in a future article.

CONCLUSION

In this article, we updated the development of ATIS as represented in the fist article, we discussed hypothesis-based research methodologies for the social sciences and found them wanting, we considered the distinction between *hypothesis* and *axiom*, we took a look at hypotheses from the social sciences, we developed an axiomatic theory from a descriptive theory and considered an application of the new theory, we considered the logic of theory development and introduced *abduction* as a means of extending theory, we proved the *Retroduction Theorem* and *Abduction Theorem*, and we described how to implement the *ATIS Option Set* to derive theory.

In future articles, the formal development of *ATIS* will be further explicated.

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